International Conference on Sciences, Engineering & Environmental Technology (ICONSEET), 10(15): 33-40, 2023

ISSN 0749-9650

www.repcomseet.org

Effects of Thermal Buoyancy Force on Fluid Flow through a Sandy soil with Time Dependent Thermal Conductivity

Olaleye O.A.* and Amola B. A.

Department of Statistics, The Federal Polytechnic, Ede, Nigeria.

*hezekiaholaleye@gmail.com (*Correspondence)*

Abstract: This study focuses on the thermal buoyancy force effects on fluid through a sandy soil (a porous medium) with time-dependent thermal conductivity. After making some basic assumptions which define the condition under which the flow took place, the continuity equation that determines the suction velocity, the energy equation which has to do with the fluid's temperature and the momentum equation which defines the velocity of the fluid's flow formed the governing equations. These were non-dimensionalized and later reduced from partial differential equation to ordinary differential equations. The analytical solution was obtained which serves as a model developed for the work. With the use of some data extracted from existing researches, the numerical results were obtained with the aid of MATLAB software. The thermal buoyancy force (Gr) alongside with magnetic field parameter (M) and permeability of the sandy soil (K) were the physical parameters examined on the flow of the fluid. The influence of these parameters was displayed on graphs. The results show that the buoyancy force has significant influence on the flow when the thermal conductivity is time-dependent.

Keywords: Magnetic field parameter, magneto-hydrodynamics, sandy soil, thermal conductivity, thermal buoyancy force.

1. Introduction

The thermal radiation may be quite significant at high operating temperatures in engineering processes, under many non-isothermal situations and in situations where convective heat transfer coefficients are small. In polymer processing industry, if the entire system involving the polymer extrusion process is placed in a thermally controlled environment, then thermal radiation effect might play an important role in controlling the heat transfer process. The knowledge of radiation heat transfer in the system can perhaps lead to a desired quality of the final product [1].

In mining industry, deeper mining is becoming more common in existing mines as the shallower parts of the ore bodies have been exhausted, but if the ore body extends at depth, then mine development must drive deeper to unlock further resources. This can be accomplished safely with proper engineering, monitoring and safety/operational procedures and policies put into place. Due to higher heat loads generated at depth, groundwater inflow temperatures increase, which present challenges from a personnel-safety standpoint as well as equipment operability and maintenance [2]. Deep mines have extra ventilation requirements. Ground temperature increases as the mine gets deeper, so dealing with increasing heat at depth becomes a significant challenge. In an effort to maintain reasonable working temperatures underground that ensure the safety of personnel, refrigeration/cooling plants must be used [2].

In their work, Akinpelu *et al* [3] compared the impacts that the solar radiation has on both the Clay-Loam and Sandy-Loam soils. By imposing a convective boundary form, they found out that solar radiation increased the temperature of both soils, though at different rates and levels.

Sharma [4] studied the unsteady free convective viscous incompressible flow past an infinite vertical porous flat plate with periodic heat and mass transfer in slip-flow regime. He discovered that the velocity increased with increasing Grashof number for both cases of water vapor and Carbon dioxide in air. He also observed that the mean Skin-friction and the amplitude of skin-friction both increased with increasing Grashof number. The temperature and concentration both increased near the plate but decreased far away from the plate.

Okedoye [5] investigated the analytical analysis of steady Magneto-hydrodynamics (MHD) free convective heat and mass transfer flow past a semi-infinite vertical porous plate in a porous medium. His results showed that the velocity profiles decreased with an increase in magnetic field parameter but increased with increase of free convection currents. The temperature and concentration profiles decreased with increase in Prandtl and Schmidt numbers respectively, but increased with increasing Dufor and Soret numbers respectively.

This work however focused on the influence of buoyancy force on the fluid flow through sandy soil with time dependent thermal conductivity.

2. Mathematical Analysis

The unsteady flow of incompressible fluid through a porous medium was studied. The three dimensional equation was reduced to an equation with function of z^* and t^* only. The flow is considered in the presence of an induced magnetic field. The medium at which the fluid flows is porous to allow the passage of the fluid. With Boussinesq's approximation, under the above conditions, the equations that govern the flow are:

Continuity equation

$$\frac{\partial w^*}{\partial z^*} = 0 \tag{1}$$

Momentum Equation

$$\frac{\partial v^*}{\partial t^*} + w^* \frac{\partial v^*}{\partial z^*} = \vartheta \frac{\partial^2 v^*}{\partial z^{*2}} + g\beta(T^* - T_\infty^*) - \frac{w}{K^*}\varphi v^* - \frac{\sigma B_0^2 v^*}{\rho} \tag{2}$$

Energy equation

$$\frac{\partial T^*}{\partial t^*} = \frac{k_0}{\rho C_p}\left((1+\alpha t)\frac{\partial^2 T^*}{\partial z^{2*}}\right) \tag{3}$$

Subject to:

$$v = V_p^* \ , \ T^* = T_w^* \text{ at the soil's surface (i.e. } z^* = 0) \tag{4}$$

$$v \to V_\infty^* \ , \ T^* \to T_\infty^* \text{ as the soil's depth increases indefinitely (i.e. } z^* \to \infty) \tag{5}$$

where z^* is the vertical axis perpendicular to the horizontal axis y^*. t^* symbolizes dimensional time, w^* stands for suction velocity. T^* signifies dimensional temperature, T_w^* and T_∞^* represent the wall and free stream temperature. C_p and ρ characterized specific heat capacity and the density. k_0 and α indicate constant thermal conductivity and thermal conductivity parameter.

Employing some dimensionless parameters in accordance to Akinpelu *et al* [3] and Nwaigwe [6],

$$t = \frac{t^* W_0^2}{w}, \quad z = \frac{W_0 z^*}{w}, \ \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \ \ v = \frac{v'}{V_\infty'} \tag{6}$$

Under a constant suction velocity as used by Mohammed [7],

$$w^* = -W_0 \tag{7}$$

Initial suction velocity is the W_0.

The momentum and energy equations (2) and (3) in non-dimensional form subsequently become:

$$\frac{\partial v}{\partial t} - \frac{\partial v}{\partial z} = \frac{\partial^2 v}{\partial z^2} + Gr\theta - \left(M + \frac{1}{K}\right)v \tag{8}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\left\{\frac{\partial}{\partial z}\left((1+\alpha t)\frac{\partial \theta}{\partial z}\right)\right\} \tag{9}$$

Subject to:

$$v = V_p \ , \ \theta = 1 \text{ at } z = 0 \tag{10}$$

$$v \to 1 \ , \ \theta \to 0 \text{ as } z \to \infty \tag{11}$$

Where the Prandtl number is given as,

$$P_r = \frac{w\rho C_p}{k_0}$$

the thermal Grashof number is,

$$Gr = \frac{wg\beta(T_w^* - T_\infty^*)}{W_0^2 V_\infty^*}$$

the Magnetic parameter is,

$$M = \frac{\sigma B_0^2 w}{\rho W_0^2}$$

the permeability of the soil type is,

$$K = \frac{K^* W_0^2}{w^2 \varphi}$$

3. Method of Solution

Following Umavathi [8], method of regular perturbation was deployed to reduced equations (8) and (9) into ordinary differential equation. This is done by assuming the solutions to be:

$$v(z,t) = v_0(z) + \varepsilon e^{i\omega t} v_1(z) + \cdots \tag{12}$$

$$\theta(z,t) = \theta_0(z) + \varepsilon e^{i\omega t} \theta_1(z) + \cdots \tag{13}$$

Higher order terms $o(\varepsilon)^2$ in equations (12) and (13) were neglected. After which the equations with their derivatives were put into equations (8) and (9), and then simplified further,

$$v_0'' + v_0' - \left(M + \frac{1}{K} \right) v_0 = -Gr\theta_0 \tag{14}$$

$$v_1'' + v_1' - \left(M + \frac{1}{K} + i\omega \right) v_1 = -Gr\theta_1 \tag{15}$$

$$(1+\alpha t) P_r^{-1} \theta_0'' = 0 \tag{16}$$

$$(1+\alpha t)\theta_1'' - i\omega P_r \theta_1 = 0 \tag{17}$$

the primes in the equations signify ordinary differentiation done with respect to z.

With the assumed solution (12) and (13), alongside their equivalent boundary conditions (10) and (11) which can be reframed as follows:

$$v_0 = V_p\,,\ v_1 = 0\,,\ \theta_0 = 1\,,\ \theta_1 = 0 \qquad \text{on} \qquad z = 0 \tag{18}$$

$$v_0 \to 1\,,\ v_1 \to 0\,,\ \theta_0 \to 0\,,\ \theta_1 \to 0 \qquad \text{as} \qquad z \to \infty \tag{19}$$

Solving equations (14) – (17) under the boundary conditions (18) and (19), the velocity and temperature distributions become:

$$v = Y_1 + Y_2 \tag{20}$$

$$\theta = 1 + Y_3 \tag{21}$$

4. Results and Discussion

The effects of the increasing and decreasing thermal buoyancy force on the fluid flow were examined using equation (20) above which is the transient velocity of the fluid. Also, the effects of an increasing magnetic field parameter and the permeability of the porous medium were also examined on the fluid flow. This was done by the use of Matlab R2009b software which generated the numerical computations. Table 1 below provides relevant thermo-physical properties of sandy soil used in the work.

Table 1: Thermo-physical properties of sandy soil

Texture Class	**Thermal Conductivity (Btu/ft hr ^{o}F)**	**Permeability (cm/hr)**
Sand	1.44	5.00

Gary [9], Soil Permeability [10]

It is also noted that some other parametric values given below were adopted in the work which are in line with some existing literatures including Olaleye *et al* [11].

$$Gr = 2.0, Pr = 0.71, M = 0.20, t = 1.0$$

All these values are valid all through the work, except or otherwise stated.

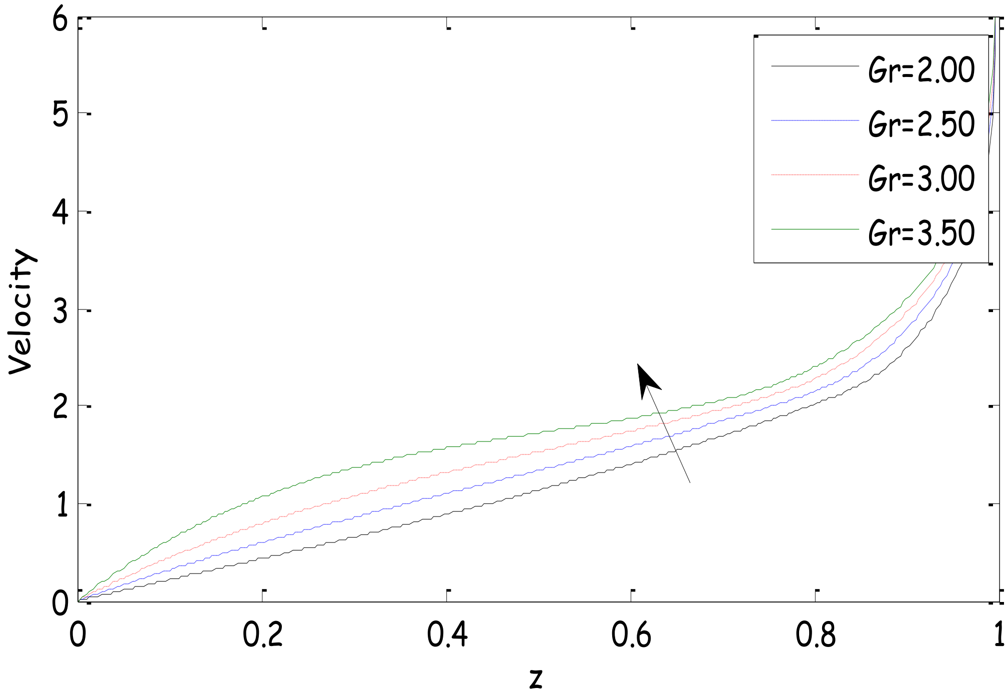

Figure 1: Effects of an increasing thermal Grashof number on the fluid flow

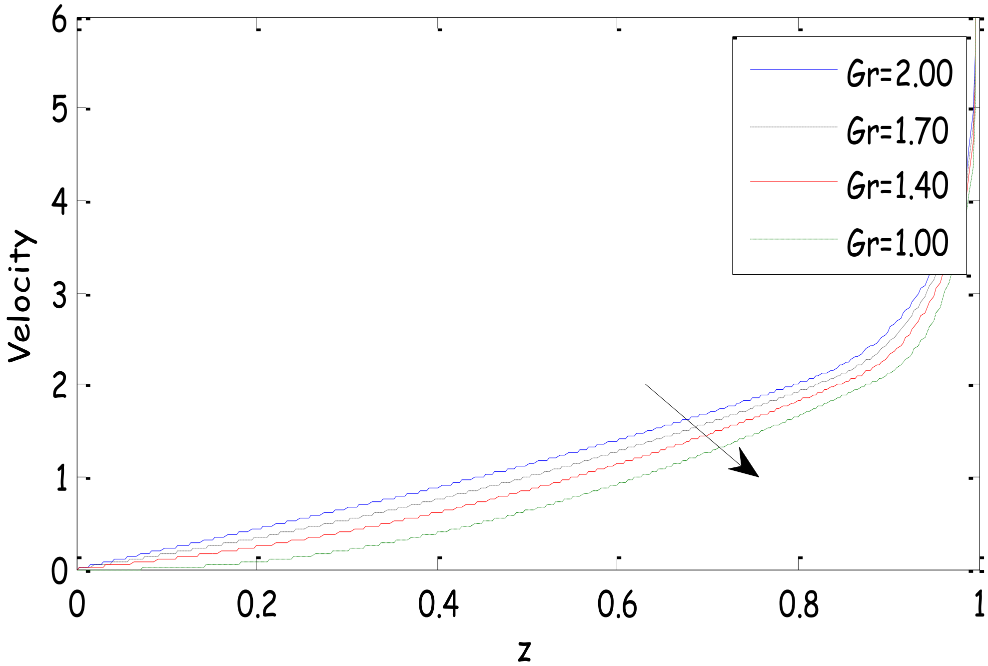

Figure 2: Effects of a decreasing thermal Grashof number on the fluid flow

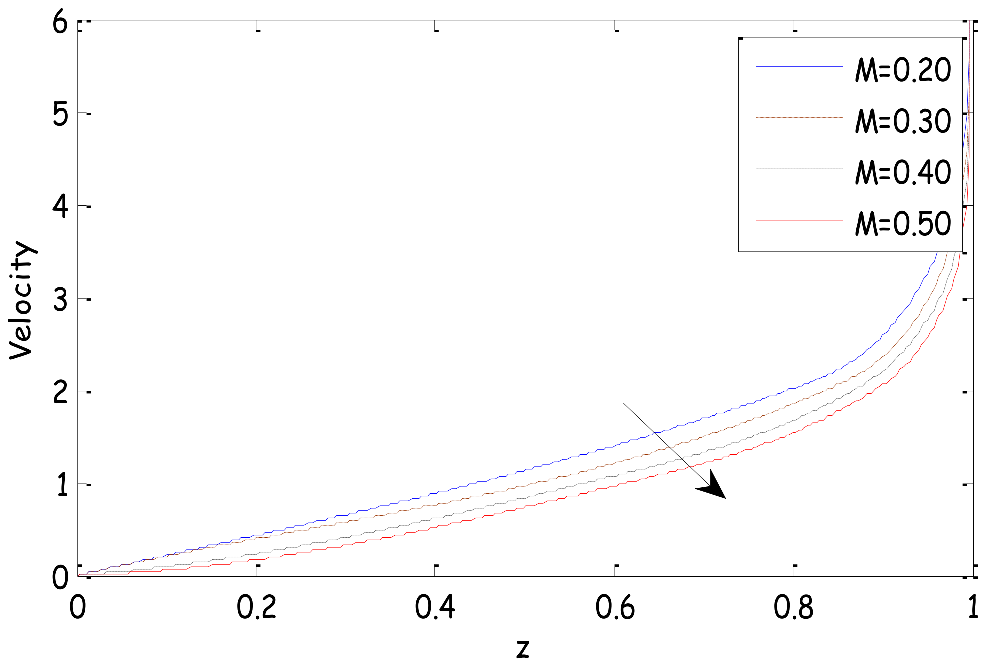

Figure 3: Effects of an increasing magnetic field parameter on the fluid flow

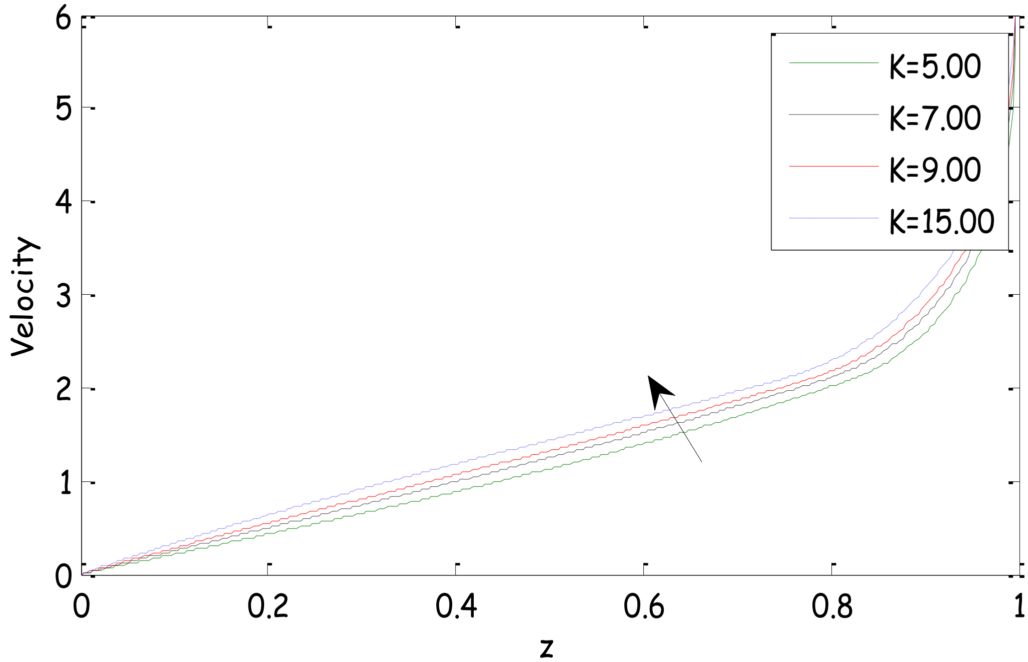

Figure 4: Effects of an increasing permeability of the porous medium on the fluid flow

Figures 1 and 2 are the effects of an increasing and decreasing thermal Grashof number on the fluid flow respectively. The results revealed that when the buoyancy force increased, the velocity of the fluid flow also is enhanced, while the decrease in the buoyancy force decreased the rate of flow.

In figure 3, the increasing magnetic field parameter obviously retards the flow of the fluid which is as a result of the Lorentz force that was generated by the magnetic field equivalent to a drag force.

Moreover, is the clearly seen according to figure 4 that when the permeability of a porous medium is increased, then the fluid has larger pores to pass through, hereby speed up the rate of flow through the medium.

5. Conclusion

The influence of thermal buoyancy force on flow through sandy soil in the presence of magnetic field with time dependent thermal conductivity was examined. It is clear to state that the thermal buoyancy force has significant effect on the fluid flow through sandy soil; an increase in the thermal buoyancy force will speed up the velocity of the fluid and vice versa.

References

[1] Abdul-Hakeem A.K., Vishnu Ganesh N. and Ganga B. (2003). *Effect of heat radiation in a Walter's liquid B fluid over a stretching sheet with non-uniform heat source/sink and elastic deformation*. Journal of King Saud University – Engineering Sciences (2014) 26, 168–175

[2] Deep-Level Mining Logistics (2014). www.miningmagazine.com, May 2014, pp. 45–47

[3] Akinpelu F. O., Olaleye O. A. and Alabison R. M. (2020). "A Comparison of the Effects of Solar Radiation on Sandy-Loam and Clay-Loam Soils with Convective Boundary Condition". *International Annals of Science*, 8(1): 130 – 137.

[4] Sharma P.K. (2005). *Fluctuating Thermal and Mass Diffusion on Unsteady Convection Flow pass a Vertical Plate in Slip-flow Regime*. Latin American Applied Research 33:313-319.

[5] Okedoye A. M. (2013). *Analytical Solution of MHD Free Convective Heat and Mass Transfer Flow in a Porous Medium*. The Pacific Journal of Science and Technology, Volume 14, Number 2.

[6] Nwaigwe, C. (2010). “Mathematical Modeling of Ground Temperature with Suction Velocity and Radiation”. *American Journal of Scientific and Industrial Research*, 238-241.

[7] Mohammed Ibrahim S. (2013). “Radiation Effects on Mass Transfer Flow through a Highly Porous Medium with Heat Generation and Chemical Reaction”. *ISRN Computational Mathematics.* Article ID 765408, 9 pages

[8] Umavathi J. C. (2015). “Combined Effects of Variable Viscosity and Variable Thermal Conductivity on Double-Diffuse Convection Flow of a Permeable Fluid in a Vertical Channel”, Transp Porous Med, **108** (2015) 659–678. DOI 10.1007/s11242-015-0494-9

[9] Gary Reysa (2015). “Ground Temperatures as a Function of Location, Season and Depth”. *Build it Solar, The Renewable Energy Site for Do-It-Yourselfers*. Sited at: www.builditsolar.com/Projects/Cooling/EarthTemperatures.htm

[10] Soil Permeability (2018). Retreaved at: http://www.fao.org/tempref/FI/CDrom/FAO_Training/FAO_Training/General/x6706e/x6706e09.htm

[11] Olaleye Olalekan A, Alabison Raimi M and Akinrinmade Victor A (2020). Effects of some physical parameters on the fluid flow through a sandy soil. International Journal of Statistics and Applied Mathematics; 5(1): 68-75

Appendix

$$Y_1 = C_1 e^{m_1 z} + C_2 e^{m_2 z} + C_3$$

$$m_1 = -\frac{1}{2} + \sqrt{\frac{1}{4} + M + \frac{1}{K}}$$

$$m_2 = -\left(\frac{1}{2} + \sqrt{\frac{1}{4} + M + \frac{1}{K}}\right)$$

$$C_1 = \frac{1 - C_3}{e^{m_1 z}}$$

$$C_2 = V_p - C_1 - C_3$$

$$C_3 = \frac{Gr}{M + \frac{1}{K}}$$

$$Y_2 = \varepsilon e^{i\omega t}\left(C_5 e^{m_4 z} + C_6 e^{m_5 z}\right)$$

$$Y_3 = \varepsilon e^{i\omega t - \left(\sqrt{\frac{i\omega P_r}{1+\alpha t}}\right)z}$$

$$m_3 = -\frac{1}{2} + \sqrt{\frac{1}{4} + i\omega + M + \frac{1}{K}}$$

$$m_4 = -\left(\frac{1}{2} + \sqrt{\frac{1}{4} + i\omega + M + \frac{1}{K}}\right)$$

$$m_5 = -\sqrt{\frac{i\omega P_r}{1+\alpha t}}$$

$$C_6 = \frac{-Gr}{m_5^2 + m_5 - \left(i\omega + M + \frac{1}{K}\right)}$$

$$C_5 = -C_6$$